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Cosmological Wave Function and Wormhole Wave Function with a Conformal Complex Scalar Field

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Quantum cosmology and the quantum wormhole with a conformal complex scalar field are discussed, the corresponding Wheeler–DeWitt equations are obtained, and the cosmological wave functions and wormhole wave functions are calculated, respectively, with different boundary conditions. From the cosmological wave function it is found that the probability density of the universe is zero at $a = 0$, while at the ground state the most probable radius is about the Planck scale. It is also shown that there exist two different types of universes, which can be connected by the quantum tunneling effect, transiting from one region to another. It follows from the wormhole wave function that the most probable radius of the wormholes is about the Planck scale, which implies that the wormhole is steady due to the quantum effect.

1. INTRODUCTION

Quantum cosmology has been widely studied (Hartle and Hawking, 1983; Vilenkin, 1982, 1988; Hawking, 1988; Duncan, 1990), applying the combination of quantum theory and gravitation theory to cosmological topics, such as: Why is our universe isotropic (Halliwell and Hawking, 1985)? Why is our observed universe a four-dimensional space–time (Wu, 1985)? What is the possible topological structure of the universe at large scale (Gibbons and Hartle, 1990)?

In this article, we discuss quantum cosmology and the quantum wormhole with a conformal complex scalar field. The Wheeler–DeWitt equations are solved for different boundary conditions, and the cosmological wave function solution and wormhole wave function solution are obtained. By

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analyzing the cosmological wave function one finds that the probability density of the universe is zero at $a = 0$ when the quantum effect is considered, while at the ground state the most probable radius of the universe is on about the Planck scale. We also show that there exist two different types of universes: (1) A baby universe appears in the range of $a \in [0, H_2^{-1} \sim l_p]$, where l_p is the Planck length; (2) a parent universe, appears when $a > H_1^{-1} \sim H^{-1}$, where H^{-1} is the Hubble radius. There exists a classically forbiden region $a \in (H_2^{-1}, H_1^{-1})$, and the baby universe and the parent universe are connected by quantum tunneling effect, transiting from one classically allowed region to another. The cosmological wave function for this model gives a more rational picture of the wormhole, and it leads to the interaction between parent and baby universes and to spatial topological transformations.

By analyzing the obtained wormhole wave function, we show that the most probable radius of the universe is on the Planck scale, which means that the wormhole is steady due to the quantum effect.

2. WHEELER–DEWITT EQUATION AND COSMOLOGICAL WAVE FUNCTION SOLUTIONS

The Robertson–Walker metric with $k = 1$ is

$$dS^2 = dt^2 - a^2(t)\, d\Omega_3^2 \tag{1}$$

where $a(t)$ is the scale factor of the universe, and

$$d\Omega_3^2 = \frac{dr^2}{1 - r^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{2}$$

The action is chosen as

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g}(R - 2\Lambda) + \int d^4x \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - \frac{1}{6} R\phi^*\phi\right) \tag{3}$$

where G is the gravitational constant, R is the curvature scalar, ϕ is the complex scalar field, and Λ is the cosmological constant.

Introducing the linear combination

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2), \qquad \phi^* = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \tag{4}$$

we obtain for Eq. (3)

$$I = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}(R - 2\Lambda) + \frac{1}{2}\int d^4x\,\sqrt{-g}[(g^{\mu\nu}\partial_\mu\phi_1\partial_\nu\phi_1 + g^{\mu\nu}\partial_\mu\phi_2\partial_\nu\phi_2) - \frac{1}{6}R(\phi_1^2 + \phi_2^2)] \tag{5}$$

Let

$$\tilde{\phi}_1 = a\phi_1, \qquad \tilde{\phi}_2 = a\phi_2, \qquad d\eta = a^{-1}\,dt \tag{6}$$

Equation (5) then becomes

$$I = \pi^2\int d\eta\,[6l_p^{-2}(-a'^2 + a^2 - \frac{\Lambda}{3}a^4) + \frac{1}{2}(\tilde{\phi}_1'^2 + \tilde{\phi}_2'^2 - \tilde{\phi}_1^2 - \tilde{\phi}_2^2)] \tag{7}$$

where the prime denotes the derivative with respect to η.

On the other hand,

$$\mathcal{L} = 6\pi^2\,l_p^{-2}(-a'^2 + a^2 - \frac{\Lambda}{3}a^4) + \frac{\pi^2}{2}(\tilde{\phi}_1'^2 + \tilde{\phi}_2'^2 - \tilde{\phi}_1^2 - \tilde{\phi}_2^2) \tag{8}$$

From the canonical quantization approach, we know that the conjugate momentums of a, $\tilde{\phi}_1$, $\tilde{\phi}_2$ are, respectively,

$$\Pi_a = \frac{\partial\mathcal{L}}{a'} = -12\pi^2 l_p^{-2}\,a' \tag{9}$$

$$\Pi_{\tilde{\phi}_1} = \frac{\partial\mathcal{L}}{\partial\tilde{\phi}_1} = \pi^2\tilde{\phi}_1' \tag{10}$$

$$\Pi_{\tilde{\phi}_2} = \frac{\partial\mathcal{L}}{\partial\tilde{\phi}_2} = \pi^2\tilde{\phi}_2' \tag{11}$$

We thus have the Hamiltonian of the system

$$H = -\frac{l_p^2}{24\pi^2}\Pi_a^2 - 6\pi^2 l_p^{-2}\,(a^2 - \frac{\Lambda}{2}a^4) + \frac{1}{2\pi^2}(\Pi_{\tilde{\phi}_1}^2 + \Pi_{\tilde{\phi}_2}^2) + \frac{\pi^2}{2}(\tilde{\phi}_1^2 + \tilde{\phi}_2^2) \tag{12}$$

Substituting operators for the momenta

$$\Pi_a \to -i\frac{\partial}{\partial a}, \qquad \Pi_{\tilde{\phi}_1} \to -i\frac{\partial}{\partial\tilde{\phi}_1}, \qquad \Pi_{\tilde{\phi}_2} \to -i\frac{\partial}{\partial\tilde{\phi}_2} \tag{13}$$

we then have the Wheeler–DeWitt equation

$$\left[\frac{l_p^2}{24\pi^2}\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a}-\frac{1}{2\pi^2}\left(\frac{\partial^2}{\partial\tilde{\phi}_1^2}+\frac{\partial^2}{\partial\tilde{\phi}_2^2}\right)\right.$$
$$\left.-6\pi^2 l_p^{-2}\left(a^2-\frac{\Lambda}{3}a^4\right)+\frac{\pi^2}{2}(\tilde{\phi}_1^2+\tilde{\phi}_2^2)\right]\psi(a,\tilde{\phi}_1,\tilde{\phi}_2)=0 \quad (14)$$

where p denotes the fuzzy of the order of operators in quantum gravitation, and since it has no significant effect on the successive discussions, we can set $p = 0$ without loss of generality.

One can find the solution to Eq. (14) with separation of variables. Setting

$$\psi(a,\tilde{\phi}_1,\tilde{\phi}_2)=F(a)G_1(\tilde{\phi}_1)G_2(\tilde{\phi}_2) \quad (15)$$

we find that Eq. (14) becomes

$$\frac{l_p^2}{24\pi^2}\frac{1}{F(a)}\frac{\partial^2 F(a)}{\partial a^2}-6\pi^2 l_p^2\left(a^2-\frac{\Lambda}{3}a^4\right)=-\lambda_0 \quad (16)$$

$$\frac{1}{2\pi^2}\frac{1}{G_1(\tilde{\phi}_1)}\frac{\partial^2 G_1(\tilde{\phi}_1)}{\partial\tilde{\phi}_1^2}-\frac{\pi^2}{2}\tilde{\phi}_1^2=-\lambda_1 \quad (17)$$

$$\frac{1}{2\pi^2}\frac{1}{G_2(\tilde{\phi}_2)}\frac{\partial^2 G_2(\tilde{\phi}_2)}{\partial\tilde{\phi}_2^2}-\frac{\pi^2}{2}\tilde{\phi}_2^2=-\lambda_0+\lambda_1 \quad (18)$$

where λ_0, λ_1 are separation constants.

Using the transformation

$$\tilde{\Phi}_1=\pi\tilde{\phi}_1,\qquad \tilde{\Phi}_2=\pi\tilde{\phi}_2,\qquad \tilde{\lambda}_1=2\lambda_1,\qquad \tilde{\lambda}_2=2(\lambda_1-\lambda_0) \quad (19)$$

we find that Eqs. (17) and (18) then become

$$\frac{\partial^2 G_1(\tilde{\Phi}_1)}{\partial\tilde{\Phi}_1^2}+(\tilde{\lambda}_1-\tilde{\Phi}_1^2)G_1(\tilde{\Phi}_1)=0 \quad (20)$$

$$\frac{\partial^2 G_2(\tilde{\Phi}_2)}{\partial\tilde{\Phi}_2^2}+(\tilde{\lambda}_2-\tilde{\Phi}_2^2)G_2(\tilde{\Phi}_2)=0 \quad (21)$$

These are standard resonance equations; accordingly,

$$\tilde{\lambda}_1=2m+1,\qquad \tilde{\lambda}_2=2n+1,\qquad m,n=0,1,2,\ldots \quad (22)$$

$$G_1(\tilde{\Phi}_1)=\exp\left(-\frac{\tilde{\Phi}_1^2}{2}\right)H_m(\tilde{\Phi}_1) \quad (23)$$

$$G_2(\tilde{\Phi}_2) = \exp\left(-\frac{\tilde{\Phi}_2^2}{2}\right) H_n(\tilde{\Phi}_2) \tag{24}$$

where H is a Hermitian polynomial.

Rearranging Eq. (16), we have

$$\frac{d^2F(a)}{da^2} - U(a)F(a) = 0 \tag{25}$$

where (see Fig. 1)

$$U(a) = 24\pi^2 l_p^{-2}\left[6\pi^2 l_p^{-2}\left(a^2 - \frac{\Lambda}{3}a^4\right) - \frac{1}{2}\lambda_0\right] \tag{26}$$

One can find the solution by the WKB approximation.

1. $U(a) < 0$, i.e., $a > H_1^{-1}$ or $0 \leq a \leq H_2^{-1}$, where

$$H_1^{-1} = \left[\frac{3 + \sqrt{9 - \Lambda l_p^2/\pi^2}}{2\Lambda}\right]^{1/2} \doteq 1.73\Lambda^{-1/2} = \sqrt{\frac{3}{\Lambda}} = H^{-1}$$

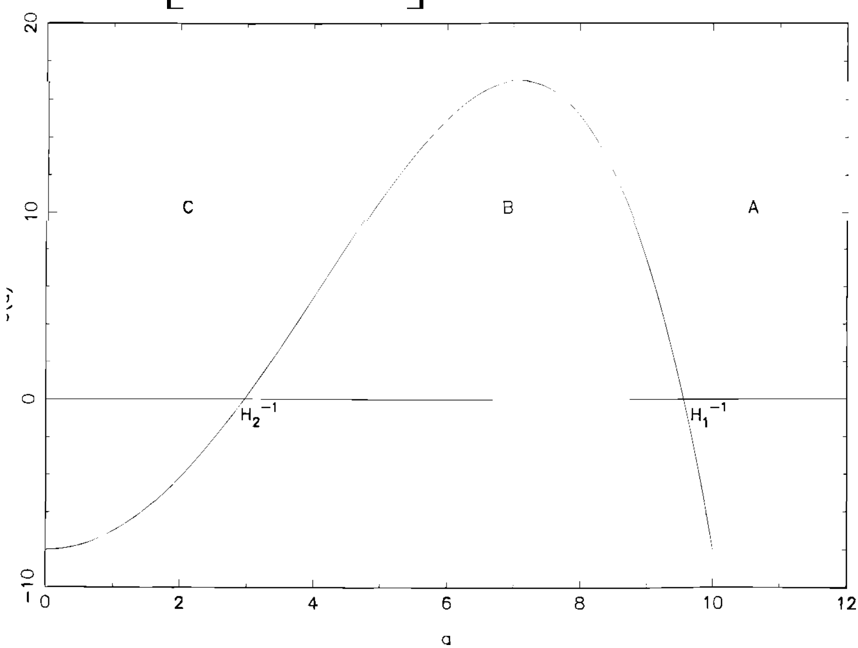

Fig. 1. The diagram for "potential" $U(a)$ versus a.

with H^{-1} the Hubble radius, and

$$H_2^{-1} = \left[\frac{3 - \sqrt{9 - \Lambda l_p^2/\pi^2}}{2\Lambda}\right]^{1/2} \doteq 0.1 l_p$$

In the case $a > H_1^{-1}$, we have

$$F_{\pm}^{(1)}(a) \doteq \frac{1}{\sqrt{P(a)}} \exp\left[\pm i \int_{H_1^{-1}}^{a} P(a')\, da' \mp \frac{\pi}{4} i\right] \tag{27}$$

where $P(a) = [-U(a)]^{1/2}$.

In the case $a < H_2^{-1}$, we have

$$F_{\pm}^{(1)}(a) \doteq \frac{1}{\sqrt{P(a)}} \exp\left[\pm i \int_{a}^{H_2^{-1}} P(a')\, da' \mp \frac{\pi}{4} i\right] \tag{28}$$

2. $U(a) > 0$, i.e., $H_2^{-1} < a < H_1^{-1}$, we then have

$$F_{\pm}^{(2)}(a) \doteq \frac{1}{\sqrt{|P(a)|}} \exp\left[\pm \int_{a}^{H_1^{-1}} |P(a')|\, da'\right] \tag{29}$$

According to Vilenkin (1988) and the connecting formulas in the WKB approach, one obtains the tunneling wave functions as follows.

1. For $a > H_1^{-1}$,

$$\psi_T^1 \doteq F_{+}^{(1)}(a) G_1(\tilde{\Phi}_1) G_2(\tilde{\Phi}_2) \tag{30}$$

2. For $H_2^{-1} < a < H_1^{-1}$,

$$\psi_T^2 \doteq F_{+}^{(2)}(a) G_1(\tilde{\Phi}_1) G_2(\tilde{\Phi}_2) \tag{31}$$

3. For $0 \leq a < H_2^{-1}$,

$$\psi_T^3 \doteq \frac{\sqrt{2}}{\sqrt{P(a)}} \exp\left(\int_{H_2}^{H_1} |P(a')|\, da'\right) \times \exp\left(i \int_{H_2}^{a} P(a')\, da'\right) G_1(\tilde{\Phi}_1) G_2(\tilde{\Phi}_2) \tag{32}$$

3. DISCUSSION OF THE COSMOLOGICAL FUNCTIONS

We know that the scale factor $a(t)$ in the Robertson–Walker metric denotes the radius of the universe; hence in the geometry described by this metric, the probability between $a \to a + da$ is

$$W(a) = 3\pi^2 a^2\, da\, \psi_T(a)\psi_T^*(a) \tag{33}$$

The probability density is given by

$$w(a) = 3\pi^2 a^2 \psi_T(a)\psi_T^*(a) \tag{34}$$

Here $\psi_T(a)$ is the value of the tunneling wave function corresponding to a, and $\psi_T^*(a)$ is the complex conjugate of $\psi_T(a)$.

We then have the following results:

1. For $a > H_1^{-1}$,

$$w(a) = \frac{3\pi^2 a^2}{P(a)} \tag{35}$$

2. For $H_2^{-1} < a < H_1^{-1}$

$$w(a) = \frac{3\pi^2 a^2}{|P(a)|} \exp\left[2 \int_a^{H_1^{-1}} |P(a')|\, da'\right] \tag{36}$$

3. For $0 \leq a \leq H_2^{-1}$

$$w(a) = \frac{6\pi^2 a^2}{P(a)} \exp\left[2 \int_{H_2}^{H_1} |P(a')|\, da'\right] \tag{37}$$

From results 1 and 2 it is clear that $a > 0$.

From result 3, when $m - n \neq 0$,

$$P(a) \overset{a \to 0}{=} |12\pi^2 l_p^{-2} \lambda_0|^{1/2} = |12\pi^2 l_p^{-2}(m - n)| \neq 0$$

When $m - n = 0$,

$$P(a) = \left| 24\pi^2 l_p^{-2} \left[6\pi^2 l_p^{-2} \left(1 - \frac{\Lambda}{3} a^2\right)\right]\right|^{1/2} a$$

We hence know from $a \to 0$ that $w(a) \to 0$.

Therefore, we conclude that the probability density of the universe is zero at $a = 0$.

Next let us consider the most probable radius (i.e. the minimum radius).

1. $a \geq H_1^{-1}$. From

$$\frac{dw(a)}{da} = 0 \tag{38}$$

we obtain

$$2(-U(a)) - \frac{1}{2}a(-U(a))' = 0 \tag{39}$$

The solution of Eq. (39) gives

$$a_p = \frac{l_p}{\sqrt{6\pi}} \sim l_p \tag{40}$$

Since $a_p < H_1^{-1}$, it follows that $a_p \notin \{a \geq H_1^{-1}\}$, that is, the minimum radius is not in the range $a \geq H_1^{-1}$.

2. $H_2^{-1} < a < H_1^{-1}$. From $dw(a)/d(a) = 0$ we obtain

$$2(U(a))^{-1/2} - 2a - \frac{1}{2}a(U(a))^{-3/2}U'(a) = 0 \tag{41}$$

i.e., $c_{14}a^{14} + c_{12}a^{12} + c_{10}a^{10} + c_8a^8 + c_6a^6 + c_4a^4 + c_2a^2 + c_0 = 0$, where

$$\begin{aligned}
c_{14} &= -192\pi^8 l_p^{-8}\Lambda^3 \\
c_{12} &= 1728\pi^8 l_p^{-8}\Lambda^2 \\
c_{10} &= -5184\pi^8 l_p^{-8}\Lambda - 144\pi^6 l_p^{-6}\Lambda^2 \\
c_8 &= 5184\pi^8 l_p^{-8} + 864\pi^6 l_p^{-6}\Lambda \\
c_6 &= -1296\pi^6 l_p^{-6} - 36\pi^4 l_p^{-4}\Lambda \\
c_4 &= 99\pi^4 l_p^{-4} \\
c_2 &= 0 \\
c_0 &= -1/4
\end{aligned} \tag{42}$$

Let $f(a) = c_{14}a^{14} + c_{12}a^{12} + c_{10}a^{10} + c_8a^8 + c_6a^6 + c_4a^4 + c_2a^2 + c_0 = 0$; then $f(0) = c_0 = -1/4 < 0$ and

$$f(0.1l_p) \approx -0.04 < 0 \tag{43}$$

$$f(l_p) = 1536\pi^8 - 576\pi^6 + 63\pi^4 > 0 \tag{44}$$

So there exists a root of Eq. (42) that is situated in $(0.1l_p, l_p)$:

$$a_p = \alpha l_p \tag{45}$$

where $\alpha \in (0.1, 1)$.

3. $0 \leq a < H_2^{-1}$

Similar to case 1, we have

$$a_p = \frac{l_p}{\sqrt{6\pi}} \sim l_p \tag{46}$$

According to the above discussions, we know that the minimum radius of the universe is of the Planck scale.

One finds the following from Eqs. (30)–(32):

Region A in Fig. 1 is a classically allowed region; the corresponding cosmological wave function concerning a is oscillating. Region B is a forbiden region; the corresponding cosmological wave function concerning a is exponentially decreasing. Region C is the same as region A. Hawking and Pagel (1990) suggested that the physical picture corresponding to the cosmological wave function in region C describes a baby universe. We thus conclude that there exist two types of universes: (1) a baby universe appears in region C; while (2) the present universe, parent universe, arises from region A. Between the two universes there exists a forbiden region (region B); the baby universe and the parent universe are connected through the quantum tunneling effect, transiting from the classically allowed region of one to that of another. The cosmological wave functions produced in such a model give a more rational picture of wormholes, which lead to interactions between the parent and baby universes.

4. WORMHOLE WAVE FUNCTION

There exist two kinds of solutions to the Wheeler–DeWitt equation (Hartle and Hawking, 1983; Hawking, 1988); one is the quantum cosmological solutions, which give rational cosmological wave functions and are identical to the evolution of the universe. The other is the quantum wormhole solutions, which take on an oscillating behavior in the classically allowed region (corresponding to $0 \to r_0 \sim l_p$ for a), and decrease exponentially in the forbbiden region (i.e., when a runs from r_0 to ∞).

For simplicity we assume $\Lambda = 0$ in Eq. (3), and set $dt = id\tau$; then the Wheeler–DeWitt equation can be written as (here we do not adopt the natural system of units)

$$\left[\frac{\hbar^2}{4\pi^4\alpha^2}\frac{1}{a^p}\frac{\partial}{\partial a}a^p\frac{\partial}{\partial a} - \alpha^2 a^2 - \frac{\hbar^2}{4\pi^4}\left(\frac{\partial^2}{\partial\Phi_1^2} + \frac{\partial^2}{\partial\Phi_2^2}\right) + (\Phi_1 + \Phi_2)\right]\psi(a, \Phi_1, \Phi_2) = 0 \tag{47}$$

Here $\alpha^2 = 3c^3/4\pi G$, and c is the speed of light.

Also, taking $p = 0$, one can obtain (by way of separation of varibles)

$$\frac{\hbar^2}{4\pi^2\alpha^2}\frac{d^2\psi(a)}{da^2} + (\lambda_0 - \alpha^2 a^2)\psi(a) = 0 \tag{48}$$

$$\frac{\hbar^2}{4\pi^2}\frac{d^2\psi(\Phi_1)}{d\Phi_1^2} + (\lambda_1 - \Phi_1^2)\psi(\Phi_1) = 0 \tag{49}$$

$$\frac{\hbar^2}{4\pi^2}\frac{d^2\psi(\Phi_2)}{d\Phi_2^2} + (\lambda_2 - \Phi_2^2)\psi(\Phi_2) = 0 \tag{50}$$

Here λ_0, λ_1 are separation constants, and $\lambda_2 = \lambda_1 - \lambda_0$.

From Eqs. (48)–(50) we obtain the rigorous solutions as follows:

$$\psi_n(a, \Phi_1, \Phi_2) = \psi_n(a)\psi_n(\Phi_1)\psi_n(\Phi_2) \tag{51}$$

w$^{\varepsilon}$re

$$\psi_n(a) = N_1 \exp\left(-\frac{1}{2}\frac{2\pi^2}{\hbar}\alpha^2 a^2\right) H_n\left(\sqrt{\frac{2\pi^2}{\hbar}}\alpha a\right) \tag{52}$$

$$\psi_n(\Phi_1) = N_2 \exp\left(-\frac{1}{2}\frac{2\pi^2}{\hbar}\Phi_1^2\right) H_n\left(\sqrt{\frac{2\pi^2}{\hbar}}\Phi_1\right) \tag{53}$$

$$\psi(\Phi_2) = \sqrt{\Phi_2} Z_{1/4}\left(i\frac{2\pi}{\hbar}\frac{\Phi_2^2}{2}\right) \tag{54}$$

Here H_n is a Hermitian polynomial, and $Z_{1/4}$ is a cylindric function.

It follows from Eq. (48) that, in the region $a^2 < \lambda_0/\alpha^2$ [$= (\lambda/\hbar)l_p^2$], the wave function is oscillating and it oscillates rapidly for large n. In the region $a^2 > \lambda_0/\alpha^2$, the wave function takes an exponential form (as far as the wave function corresponding to the WKB approximation is concerned), and therefore the wave functions obtained in this article are wormhole wave functions describing quantum wormholes.

In the geometry described by the Robertson–Walker metric, the probability of the wormhole situated between $a \to a + da$ is

$$W_n(a) = 3\pi^2 a^2\, da\, \psi_n(a)\psi_n^*(a) \tag{55}$$

while the probability density is

$$w_n(a) = 3\pi^2 a^2 \psi_n(a)\psi_n^*(a) \tag{56}$$

Then

$$w_n(a) \overset{a \to 0}{\to} 0 \tag{57}$$

which has a simple interpretation: for an arbitrary nth quantum state, near $a = 0$, the probability of the baby universes and wormholes is zero.

For a ground state ($n = 0$) one has

$$w_0(a) = 3\pi^2 N_1 \exp\left(-\frac{2\pi^2}{l_p^2} a^2\right) a^2 \tag{58}$$

where $l_p^2 = (\hbar/\alpha^2) = (4\pi G\hbar/3c^3)$.

The position of the maximum probability can be determined by

$$\frac{dw_0}{da} = 0 \tag{59}$$

i.e.,

$$3\pi^2 N_1 \exp\left(-\frac{2\pi^2}{l_p^2} a^2\right) 2a \left[1 - \frac{2\pi^2}{l_p^2} a^2\right] = 0 \tag{60}$$

Since $a \neq 0$, we obtain

$$a = \frac{l_p}{\sqrt{2}\pi} \tag{61}$$

This implies that the most probable radius of the wormholes is of the Planck scale, namely, the quantum effect can make a wormhole survive gravitational collapse and remain steady.

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